Correlation of Minimum Fluidization, Fluidization and Terminal Velocities in the Design of Fluidized Bed Electrochemical Reactor

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Abstract
The separation of mixture in phases or waste treatment is optimally successful using fluidized bed reactor, which is operated in stages of increasing velocities ranging from minimum fluidization velocity, fluidization velocity and terminal velocity that can cause flow of the fluid medium. A design based on the first principles of mass transfer is carried out on fluidized bed electrochemical reactor operating under Current Limit conditions to investigate the correlation of minimum fluidization velocity, fluidization velocity and terminal velocity. The design was based on the treatment of 50,000m^3/year of industrial wastewater and the recovery of copper. The minimum fluidization velocity was 0.03 m/s, fluidization velocity was 0.07m/s and terminal velocity was 1.53m/s which agrees with Hartog et al., (2008) and Roland et al., (1999) which stated symbolically that $U_{mf}<U_f<U_t$. The reactor volume was calculated to be 1.42m^3 while the reactor height was 1.81m. The resident time was 736 second, and the space velocity was 0.0014 sec^{-1}, while the corresponding volumetric flow rate was 0.00193m^3/s at 95% conversion. The sphericity of the particles was 0.6 and the voidage (porosity) was 0.5. The flow regime was turbulent with pressure drop of 272.23N/m^2. The over potential of the system was 0.70V, while the required power was 0.5254Nm/s and the mass of catalyst (Zeolite A) was 61.07kg respectively. These results can be used for Engineering scale-up of this type of process.

Keyword: *Correlation, Fluidized Bed Electrochemical Reactor, Current Limit, Minimum Fluidization Velocity, Fluidization Velocity, Terminal Velocity, Industrial Wastewater Treatment*

1.0 Introduction

The operational activities of any chemical processing industry in the transfer or treatment of fluids in mixed phases can be done optimally using fluidized bed reactor. This reactor is operated in stages of velocities that can cause flow of the fluid medium of which one in the mixture is fluidized. It may be fluidized for separation or waste treatment which is not usually purged openly but contained to be discharged after treatment to reduce the degree of toxicity in relation to its concentration.
In the selection of the type of reactor that can perform in the industrial wastewater treatment process, fluidized bed reactor becomes the more acceptable one because of its functionality and characteristics. In the last few decades, fluidized bed reactor had wide application in wastewater treatment as compared to the conventional reactors (Avarado-Lassman, 2008).

In fluidized bed reactor, the solid particles are supported by an upward flow of fluidizing fluid. The conversion of fluidized bed reactor electrochemically, makes it more efficient in the treatment process. It becomes more convenient, safer, less time-consuming and cheaper among other appreciable characteristics. It is a combination of the two most common, packed-beds and stirred tank, continuous flow reactors. It is very important to chemical engineering because of its excellent heat and mass transfer characteristics.

Fluidization occurs when small solid particles are suspended in an upward-flowing stream of fluid. The fluid velocity is sufficient to suspend the particles, but not large enough to carry them out of the vessel (Brown and Fogler, 2008). The solid particles swirl around the bed rapidly, creating excellent mixing among them. The solids or catalytic particles are supported by an up flow of fluidizing fluid (Suleiman *et al*., 2013).

Because air flows upward through the filter (distributor), the sand in the filter becomes suspended or fluidized in the air column, forming a fluidized bed of sand. If the flow of air is controlled properly, the sand does not flow out of the filter, but remains suspended (Doki, 2011). This is because the flow of air is fast enough to keep the sand in suspension. The weight of the sand prevents it from escaping the column or reactor. Fluidized bed filters are self-cleaning, and require little or no maintenance.

Fluidized bed cell designs, therefore, provide dynamic conditions that are believed to be important for the observation of excess enthalpy in cold fusion experiments (Miles, 2003).
The most typical fluidized bed reactor consists of a bed of great height, whose lower part introduces water, through a distribution system, at a high enough velocity to fluidize or expand the bed (Alfredo *et al*., 2014). According to Ehirim, (2012) fluidized bed electrodes consisting of electrically conducting particles in an electrolytic solution behave as tridimensional. He stated further that they are frequently employed for the projects of electrochemical reactors; and that for some applications, these electrodes, due to their high surface areas, are usually preferred to other porous electrodes because they offer high rate of mass transfer.

Mean diameter of the bed particles, sphericity of the particle, void fraction of the bed, minimum fluidization velocity, maximum fluidization velocity and terminal settling velocity are the parameters necessary to a fluidized bed reactor for wastewater treatment (Farhana *et al*., 2014).
Normal measured sphericity values for a typical granular solid range from 0.5 to 1, with 0.6 being a choice for every round shaped particle (Farhana *et al*., 2014). The average sphericity for the particle mixture can be calculated by two different methods. First, by the use of the correlation of Narsimham for mono disperse particles (Narsimham, 1965). In the second method, the average sphericity has been calculated from the sphericity data of irregular particles of dolomite of different sizes reported by (Singh, 1997).

This assumption of homogeneous behavior for the liquid–solid fluidization systems considers the liquid–solid fluidization as an ideal system and forms the basis of Richardson and Zaki and Kwauk's work (Lee, 2000). Minimum fluidization velocity is for fluidizing the bed particles from the bed. It is the velocity required to begin the fluidization at which the weight of particles gravitational force equals the drag on the particles from the rising gas (Liang *et al.*, 1996).

In most of gas fluidized beds applications, bubbles appear when the superficial gas velocity exceeds the minimum velocity required to overcome the weight of the bed (Almendros-Ibáñez, 2010). If gas or liquid velocity is increased to a sufficient limit, that the drag on every particle will surpass the gravitational force on the particles. This velocity is called Maximum fluidization velocity. Maximum fluidization is important parameter to know for avoiding particle entrainment. The operating fluidization velocity depends on the maximum fluidization velocity too.

Correlations for calculating maximum fluidization velocity can be found from (Bourgeois *et al*., 1968) and (Lu *et al*., 2004). The solid particles are entrained when the upward velocity of fluid is sufficiently high and thus they are carried up with the fluid. At this point elutriation occurs. This velocity is called elutriation velocity. The minimum elutriation velocity for particles of a given size is the velocity at incipient entrainment, and is assumed to be equal to the terminal velocity, U_t (Suleiman *et al*., 2013). Elutriation is the selective removal of solid particles by entrainment on the basis of size (Roland *et al.*, 1999). Proper fluidization occurs at a velocity called actual fluidized velocity U_f and the relationship between the minimum fluidized velocity U_{mf} and terminal velocity U_t is given by $U_{mf}<U_f<U_t$ (Hartog *et al.*, 2008 and Roland *et al.*, 1999). If the gas velocity is increased to a sufficiently high value, however, the drag on an individual particle will surpass the gravitational force on the particle, and the particle will be entrained in a gas and carried out of the bed. The point at which the drag on an individual particle is about to exceed the gravitational force exerted on it is called the ***maximum fluidization velocity*** (Suleiman *et al*., 2013).

2.0 Development of Design Equations (Fluidized Bed)

The design equations are developed for the computation of the functional parameters of Fluidized bed electrochemical reactor for the treatment of industrial wastewater and recovery of copper using Zeolite A as the bed particles and as catalyst.

For the development of the design equation, the following considerations were made.

1. Only one component k = 1 reacts in the system.
2. There is no accumulation of the chemical specie(s) in the liquid phase.
3. Hydrodynamic and electrochemical operating conditions are maintained constant.
4. There are no concentration and temperature gradients in the reactor.
5. The reactor operates as a CSTR.

2.1 Material Balance

$$\{Rate\ of\ \inf low\} = \{Rate\ of\ outflow\} + \{Rate\ of\ Disappearance\} + \{Accumulation\} \quad (1)$$

Rate of Inflow = F_{A0} (2)

Rate of Outflow = $F_A = F_{A0}\,(1-X_A)$ (3)

Rate of disappearance = $(-r_A)V_R$ (4)

Rate of accumulation = $\frac{dN_A}{dt}$ (5)

Substituting equations (2) through (5) into equation (1)

$$F_{A0} = F_A + (-r_A)V_R + \frac{dN_A}{dt} \quad (6)$$

But the rate of accumulation for CSTR, $\frac{dN_A}{dt} = 0$

Hence,

$$F_{A0} - F_A = (-r_A)V_R \quad (7)$$

But

$$F_A = F_{A0}(1-X_A) \quad (8)$$

Substitution of equation (8) in equation (7) gives:

$$F_{A0}X_A = (-r_A)V_R \tag{9}$$

$$V_R = \frac{F_{A0}X_A}{-r_A} \tag{10}$$

But the rate term in equation (10) can be compared to equation (13). That is

$$-r_A = -R_{k,s} = a_m \frac{(1-\varepsilon)}{\varepsilon}\frac{D_K}{\delta}(C_{k,s} - C^*_{k,s})$$

Hence, $V_R = \dfrac{F_{A0}X_A}{a_m \frac{(1-\varepsilon)}{\varepsilon}\frac{D_K}{\delta}(C_{k,s} - C^*_{k,s})}$

$$C^*_{k,s} = C_{k,s}(1 - X_A)$$

$$V_R = \frac{F_{A0}X_A}{a_m \frac{(1-\varepsilon)}{\varepsilon}\frac{D_K}{\delta}\left(C_{k,s} - C_{k,s}(1 - X_A)\right)} \tag{11}$$

2.2 The design Equations for other Functional Parameters of the reactor.

i) **Reactor Height**.

The height of cylindrical shaped reactor is given as

$$H_R = \frac{4V_R}{\pi d^2} \tag{12}$$

or

$$H_R = \frac{4F_{A0}X_A}{\pi d^2 a_m \frac{(1-\varepsilon)}{\varepsilon}\frac{D_K}{\delta}\left(C_{k,s} - C_{k,s}(1 - X_A)\right)} \tag{13}$$

ii). Space Time (τ)

$$\tau = \frac{V}{V_o} \tag{14}$$

iii) Space Velocity (S_V)

$$S_v = \frac{V_0}{V} = \tau^{-1} \tag{15}$$

iv) Mass of Catalyst (M)

The mass of catalyst can be determined using the equation stated by Brown and Fogler (2008).

$$W_c = \rho_c A_c h(1-\varepsilon) \tag{16}$$

v) Pressure Drop (ΔP)

The pressure drop along fluidized bed reactor in terms of both laminar and turbulent flows is expressed using Ergun equation (Idris *et al*, 2007) as stated below:

Laminar or Streamline Flow

$$\Delta P = \left[150\frac{(1-\varepsilon)^2}{\varepsilon^3}\frac{\mu u}{d_p^2} + 1.75\frac{(1-\varepsilon)}{\varepsilon^3}\frac{u\rho_f}{d_p}\right]\times h$$

(17)

Turbulent Flow

$$\Delta P = [(\rho_s - \rho_f)(1-\varepsilon)g]\times h$$

(18)

vi) Minimum Fluidized Velocity (U_{mf})

General Equation for the Fluid, Solid and Bed

$$u_{mf}^2 + \frac{150(1-\varepsilon_{mf})\mu_f}{1.75\rho_f d_p^1}u_{mf} - \frac{g(\rho_p-\rho_f)\varepsilon_{mf}^3 d_p^1}{1.75\rho_f} = 0$$

(19)

vii) Relatively Small Particle Size and Small Reynolds Number

For relatively small particle size as well as small Reynolds number and relatively large particle size as well as large Reynolds number, the minimum fluidization velocity is expressed by Roland *et al* (1999) as:

$$u_{mf} = \frac{8(\rho_p-\rho_f)(d_p^1)^2}{150(1-\varepsilon_{mf})/\varepsilon_{mf}^3}$$

(20)

viii) Relatively Large Particle Size and Large Reynolds Number

$$u_{mf} = \left[\frac{g(\rho_p-\rho_f)\varepsilon_{mf}^3 d_p^1}{1.75\rho_f}\right]^{1/2}$$

(21)

ix) Reynolds Number

$$\mathrm{Re} = \frac{\rho_f d_t u_f}{\mu}$$

(22)

x) Fluidized Velocity (U_f)

This is the velocity at which fluidization occurs. It can be expressed using Kozeny-Carmen equation as stated under:

$$u_f = \frac{(\rho_p-\rho_f)gd_p^2}{150\mu}\frac{\varepsilon^3}{1-\varepsilon}$$

(23)

xi) Terminal Velocity (U_t)

Terminal velocity can be assessed by application of the Richardson–Zaki equation (Richardson & Zaki (1954)), which describes expansion of the fluidized bed as:

$$U = U_t \varepsilon^n \tag{24}$$

xii) Sphericity

It can be expressed using Narsimhan's correlation as stated below.

$$\left(\frac{1-\varepsilon}{\varphi_s}\right) = 0.231 \log d_{psm} + 1.417$$

$$\varphi_s = \frac{1-\varepsilon}{0.231 \log d_{psm} + 1.417} \tag{25}$$

xiii) Power Requirement

It is represented by the equation below according to Suleiman *et al.*, (2013).

$$P = \Delta P * V_0 \tag{26}$$

3.0 Design Calculations

The basis of this design was on the treatment of 50,000m^3/year of industrial wastewater vis-à-vis recovery of copper. The prime calculations are shown in Appendix A.

Table 2.1 Data for the determination of the process parameters

Variable	Value
Density of Aqueous $CuSO_4$ [ρ] g/m^3	3600000
Mass of CuSO4 solution [M] g/mol	159.609
Input Mole Flow rate [F_{A0}] mol/s	43.53
Input Mass Flow rate [F_{A0}] g/s	6948
Output Mass Flow Rate [F_A] g/s	347.40
Initial Concentration [C_{A0}] g/m^3	508.358
Specific surface area [a_m] m^{-1}	6000
Porosity [ε] -	0.5
Diffusion Coefficient [D_k] m^2/s	1.67E-7
Bed width [δ] m	0.1E-3

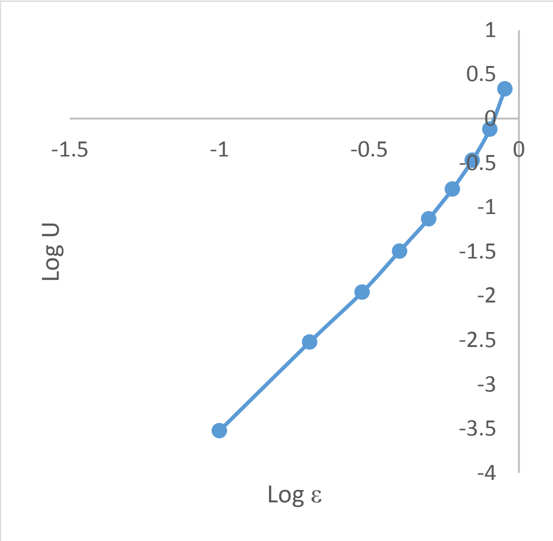
4.0 Results and Discussion

Table 4.1 Calculated Values of some of the Functional Parameters for 50,000m^3/year waste treatment using Electrochemical (Fluidized bed) Reactor.

Vol. Flow rate [m^3/s]	Reactor Volume [m^3]	Reactor Height [m]	Residence Time [s]	Space Velocity [s^{-s}]	Catalyst Mass [kg]	Minimum Fluidized Velocity [m/s]	Fluidized Velocity [m/s]	Terminal Velocity [m/s]	Reynolds Number [-]
0.00193	1.42	1.18	736	0.0014	61.07	0.03	0.07	1.53	91503.25

Table 4.1 shows the design values of the functional parameters for the treatment of a 50,000m^3/year of wastewater. These values are in close agreement with those found in literatures (Gouhua, 2004).

4.1 Result



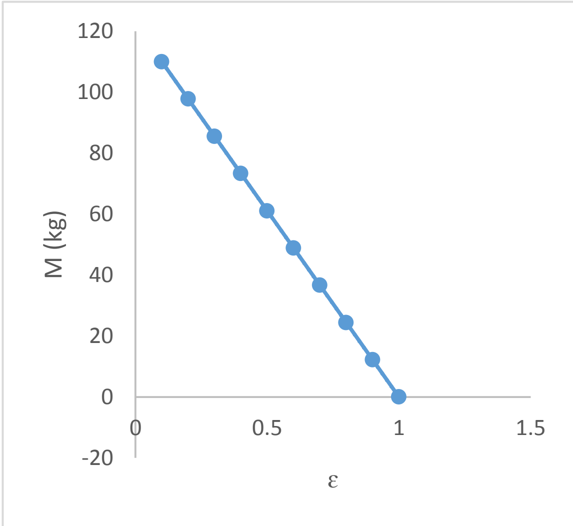
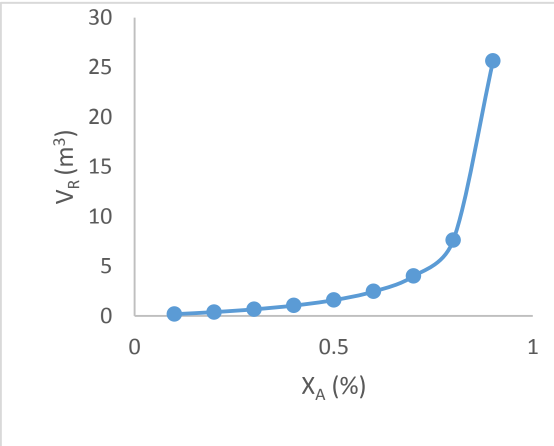


Figure 3.1: Effect of Fluidized Velocity on Bed Voidage Figure

3.2: Effect of Catalyst Mass on Bed Voidage



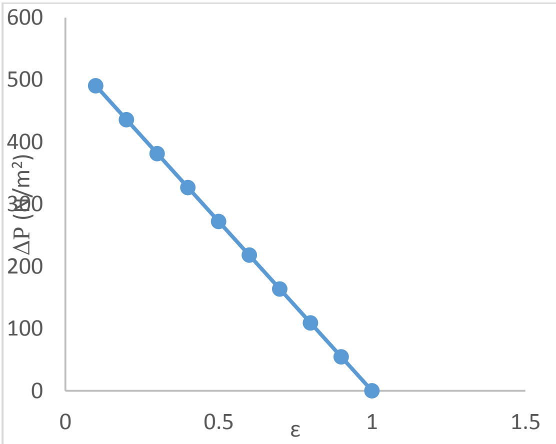
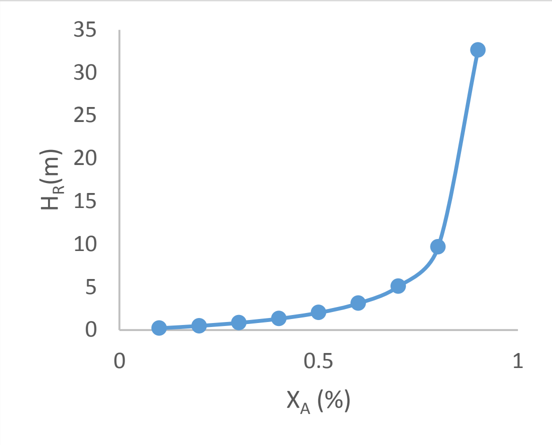


Figure 3.3: Effect of Reactor Volume on Conversion

Figure 3.4: Effect of Pressure Drop on Bed Voidage



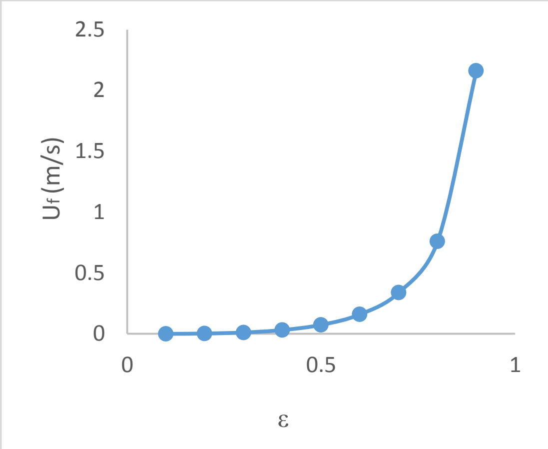


Figure 3.5: Effect of Reactor height on Conversion Figure **3.6: Effect of Fluidized Velocity on Bed Voidage**

3.2 Discussion

Figure 3.1 shows the relationship between fluidized velocity and voidage or porosity where the log-log plot of the parameters is plotted. It shows a linear relationship among them. This graph is analytically used to determine the terminal velocity based on equation 2.24.
Figure 3.2 is a graph plotting the relationship between catalyst mass and voidage (porosity). It shows an inverse relationship on each other which implies that as catalyst mass increases, the voidage (porosity) decreases and vice-versa.

Figures 3.3 and 3.5 are graphs showing the relationship between fractional conversion (yield) and the reactor volume as well as the height. They both show the same trend indicating that the reactor volume and height increase as the conversion does.
Figure 3.4 shows the effect of pressure drop on voidage. It indicates that increase in pressure drop is as a result of decrease in the voidage. When the voidage is high, the fluid flows freely without experiencing pressure drop, but as the voidage decreases, definitely, pressure drop is noticed in the fluidizing system.
Figure 3.6 shows the relationship between fluidized velocity and voidage or porosity. It shows a linear relationship among them such that as the fluidized velocity increases the voidage also increases.

4.0 Conclusion

The fluidized bed reactor was designed for the treatment of 50,000 m^3/year of industrial wastewater and recovery of copper. The reactor volume is calculated to be 1.42m^3 while the reactor height is 1.81m and the volumetric flow rate is calculated as 0.00193m^3/s at 95% conversion. The resident time is 736 second, and the space velocity is 0.0014 per second. The minimum fluidization, fluidization and terminal velocities are 0.03, 0.07 and 1.53m/s following a serial trend of increase in agreement with literature at voidage value of 0.5. The pressure drop is 272.23N/m^2 in the turbulent flow regime. The mass of catalyst and surtension are calculated as 61.07kg and 0.70V. The design calculation is useful for Engineering Scale-up.

Nomenclature

a_m	Specific superficial area of the solid (m^{-1})
$C_{k,s}$	Concentration of chemical specie k in the liquid phase (g/m^3)
$C^*_{k,s}$	Concentration of chemical specie in the liquid phase at the electrode surface

	(g/m^3)
D_k	Diffusion coefficient of the chemical specie k (m^2/s)
E_{eq}	Equilibrium potential (V)
F	Faraday Constant (96500C/mol)
M	Particle mass (g)
n	Number of electrons
$R_{k,s}$	Rate of reaction in terms of mass of chemical specie k per unit volume of the liquid phase (g/m^3s)
R	Universal gas constant (8.314 $J.mol^{-1}K^{-1}$)
T	Temperature (K)
η	Overpotential (V)
ϕ_m	Solid phase potential (V)
ϕ_s	Liquid phase potential (V)
σ_s	Conductivity of the liquid phase ($\Omega^{-1}m^{-1}$)
σ_m	Conductivity of the matrix ($\Omega^{-1}m^{-1}$)
V_R	Reactor volume (m^3)
H_R	Reactor Height (m)
V_0	Volumetric flow rate (m^3/s)
X_A	Conversion (%)
F_A	Rate of Outflow (g/s)
F_{A0}	Rate of feed inflow (g/s)
d	Reactor diameter (m)
τ	Resident time (s)
S_V	Space Velocity (m/s)
W_c	Mass of catalyst (kg)
ρ_c	Density of catalyst (kg/m^3)
A_c	Column cross-section area (m^2)
ΔP	Pressure drop (N/m^2)
h	Height of the bed (m)
μ	Fluid viscosity (kg/s.m)
ρ_f	Fluid density (kg/m^3)
ε	Void fraction of bed
d_p	Particle diameter (m)
ρ_s	solid density (kg/m^3)
g	Acceleration due to gravity (m/s^2)
U_{mf}	Minimum fluidized velocity (m/s)
U_f	Fluidized velocity (m/s)
U_t	Terminal velocity (m/s)
Re	Reynolds number (-)
φ	Spericity (-)
P	Pump power (Nm/s)

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APPENDIX A

Volumetric Flowrate

The treatment of 50,000m^3/year was based on 300 days out of 365 days in the year. This is because routine maintenance and other operational factors were taken into account. Thus, the reactor volumetric flowrate required for the treatment was obtained as stated below:

$$V_0 = \frac{50{,}000m^3}{300days} \times \frac{1day}{1440\min} \times \frac{1\min}{60\sec} = 0.00193m^3/\sec$$

Input Mass flow rate, $\overset{o}{m} = Density, \rho \times Volumetric\ \ flow\, rate, \overset{o}{V}$

Density of aqueous $CuSO_4$ = 3600000g/m^3

Molar mass of aqueous $CuSO_4$ = 159.609 g/mol

$$\overset{o}{m} = 3600000g/m^3 \times 0.00193m^3/s = 6948g/s$$

Input Mole flow rate, $F_{A0} = \frac{Mass\ flow\, rate}{Molar\ mass} = \frac{6948}{159.609} = 43.53mol/s$

Input Mass flow rate, F_{A0} = Input Mole flow rate X Molar mass
= 43.53 X 159.609 = 6948 g/s

$F_A = F_{A0}(1\text{-}X_A)$

Where:

F_A = Output Mole flow rate (mol/s)

X_A = Conversion (fraction) = 95% = 0.95

F_A = 43.53 (1-0.95) = 2.1765 mol/s

Output Mass flow rate = Output Mole flow rate X Molar mass
F_A = 2.1765 X 159.609 = 347.40 g/s

$C_A = C_{A0}(1\text{-} X_A)$

Where:

$C_{A0} = C_{k,s}$ = 508.358 g/m^3

$C_A = C^*_{k,s}$ = 508.358 (1-0.95) = 25.42 g/m^3

Data Feed

F_{A0} = 43.53 mol/s = 6948 g/s

V_0 = 0.00193 m^3/s

$C_{A0} = C_{k,s}$ = 508.358 g/m^3

Conversion, X_A = 0.95 = 95%

Reactor Volume

The reactor volume was calculated using the general formula below according to equation 11:

$$V_R = \frac{F_{A0}X_A}{a_m \frac{(1-\varepsilon)}{\varepsilon}\frac{D_K}{\delta}\left(C_{k,s} - C_{k,s}\left(1 - X_A\right)\right)}$$

Where:

F_{A0} = 6948g/s

X_A = 0.95

a_m = 6000 m^{-1}

ε = 0.5

D_k = 1.6 x 10^{-7} m^2/s

δ = 0.1 x 10^{-3} m

$$C_{k,s} = 508.358 g/m^3$$

$$C^*_{k,s} = 25.42 g/m^3$$

$$V_R = \frac{F_{A0} X_A}{a_m \frac{(1-\varepsilon)}{\varepsilon} \frac{D_K}{\delta} \left(C_{k,s} - C_{k,s}(1 - X_A) \right)}$$

$$V_R = \frac{6948 \times 0.95}{6000 \frac{(1-0.5)}{0.5} \frac{1.6 \times 10^{-7}}{0.1 \times 10^{-3}} (508.358 - 508.358(1-0.95))} = 1.42 m^3$$

Reactor Height

The reactor height was calculated according to equation 13

$$H_R = \frac{4 F_{A0} X_A}{\pi d^2 a_m \frac{(1-\varepsilon)}{\varepsilon} \frac{D_K}{\delta} \left(C_{k,s} - C_{k,s}(1 - X_A) \right)}$$

Where:

V_R = 1.42 m
d = 1m
F_{A0} = 6948g/s
X_A = 0.95
a_m = 6000m^{-1}
ε = 0.5
D_k = 1.6 x 10^{-7} m^2/s
δ = 0.1 x 10^{-3}m

$$C_{k,s} = 508.358 g/m^3$$

$$C^*_{k,s} = 25.42 g/m^3$$

$$H_R = \frac{4 \times 6948 \times 0.95}{3.142 \times 1^2 \times 6000 \frac{(1-0.5)}{0.5} \frac{1.6 \times 10^{-7}}{0.1 \times 10^{-3}} (508.358 - 508.358(1-0.95))} = 1.81 m$$

Residence Time (τ)

The residence time was calculated according to equation 14

$$\tau = \frac{V}{V_0}$$

Where:

V_R = 1.42 m^3
V_0 = 0.00193 m^3/sec

$$\tau = \frac{V}{V_0} = \frac{1.42}{0.00193} = 736 \sec$$

Space Velocity (S_v)

Space very was calculated using equation 15 as:

$$S_v = \frac{V_0}{V} = \tau^{-1} = 736^{-1} = 0.0014 \sec^{-1}$$

Catalyst Mass (M)

From equation 16, the catalyst mass was calculated thus:

$$W_c = \rho_c A_c h(1-\varepsilon)$$

Where:

W_c = mass of catalyst (kg)

ε= 0.5

ρ_c = density of the catalyst = 1555kg/m^3

A_c = Column cross-section area (m)

h = Height of the bed = 0.10m

But,

$$A_c = \pi r^2$$

$$r = 0.5d$$

d = Bed diameter = 1.0m

$$A_c = 3.142 \times (0.5)^2 = 0.7855m^2$$

$$W_c = \rho_c A_c h(1-\varepsilon)$$

$$W_c = 1555 \times 0.7855 \times 0.10(1-0.5) = 61.07kg$$

Calculation of Minimum Fluidized Velocity (u_{mf})

The minimum velocity was calculated using equation 21

$$u_{mf} = \left[\frac{g(\rho_p - \rho_f)\varepsilon_{mf}^3 d_p}{1.75\rho_f}\right]^{1/2}$$

$$u_{mf} = \left[\frac{9.81 \times (1555-1000) \times (0.5)^3 \times 2.5 \times 10^{-3}}{1.75 \times 1000}\right]^{1/2} = 0.03m/s$$

Calculation of fluidized velocity (u_f)

From equation 23, the fluidized velocity was calculated thus:

$$u_f = \frac{(\rho_p - \rho_f)gd_p^2}{150\mu}\frac{\varepsilon^3}{1-\varepsilon}$$

$$u_f = \frac{(1555-1000) \times 9.81 \times (2.5 \times 10^{-3})^2}{150 \times 7.65 \times 10^{-4}}\frac{(0.5)^3}{(1-0.5)} = 0.07m/s$$

Terminal Velocity

This can be calculated by using equation 24:

$$U = U_t \varepsilon^n$$

$$LogU = LogU_t + Log\varepsilon^n$$

$$LogU = LogU_t + nLog\varepsilon$$

From figure 6, the equation of the straight line which is analogous to $LogU = LogU_t + nLog\varepsilon$ is $y = 3.8775x + 0.1845$

From this relationship, it can be seen that:

n = 3.8775

$Log\ U_t$= 0.1845

$U_t = 10^{0.1845} = 1.53$m/s

Sphericity

The sphericity of the particles was calculated using equation 25 as stated below.

$$\varphi_s = \frac{1-\varepsilon}{0.231 \log d_{psm} + 1.417}$$

$$\varphi_s = \frac{1-0.5}{0.231 \log(2\times10^{-3}) + 1.417}$$

$$\varphi_s = \frac{0.5}{0.8159} = 0.6$$

Reynolds Number (Re)

The Reynolds number is calculated using equation 22 as stated below.

$$\text{Re} = \frac{\rho_f d_t u_f}{\mu}$$

Where:

ρ_f = Fluid density = 1000kg/m^3

d_t= Column diameter = 1m

U_f = Fluidized velocity = 0.07m/s

μ = 7.65× 10^{-4} Kg/m.s

$$\text{Re} = \frac{1000\times1\times0.07}{7.65\times10^{-4}} = 91503.27$$

Pressure Drop (ΔP)

Assuming laminar flow and calculating according to equation 17:

$$\Delta P = \left[150\frac{(1-0.5)^2}{(0.5)^3}\frac{7.65\times10^{-4}\times0.07}{(2.5\times10^{-3})^2} + 1.75\frac{(1-0.5)}{(0.5)^3}\frac{0.5\times1000}{2.5\times10^{-3}}\right]\times0.1 = 19856.04 N/m^2$$

Assuming turbulent flow and calculating according to equation 18:

$$\Delta P = [(1555-1000)(1-0.5)\times9.81]\times0.1 = 272.23 N/m^2$$

Power Requirement

The required operational power was calculated using equation 26.

$$P = \Delta P * V_0$$

Considering turbulent flow which has a lower pressure drop,

$$P = 272.23\times0.00193 = 0.5254 Nm/\sec$$

Considering laminar flow which has a higher pressure drop,

$$P = 19857.04\times0.00193 = 38.3241\ Nm/\sec$$

Overpotential (η)

The system overpotential was calculated using equation 18,

$$\eta = \phi_m - \phi_s - \left[E_{eq} + \frac{RT}{nF}\ln(C^*_{k,s})\right]$$

$$\eta = 3 - 1.5 - \left[0.76 + \frac{8.314\times303}{2\times96500}\ln(25.42)\right] = 0.70V$$